System Response (F=MA)

An understanding of how a spring mass system responds to vibratory influences is helpful in understanding, recognizing and solving many problems encountered in vibration measurements. In this application note the combined effects of system mass, stiffness, and damping properties are presented to reveal the cause and characteristics of resonance.

All machines have three fundamental traits, which combine to determine how the machine will react to excitation forces. These traits are stiffness K, damping D, and mass M. These traits actually represent forces inherent to every machine and structure, tend to resist or oppose vibration.

From an analysis standpoint, it should be remembered that machines, along with their supporting structures, are complex systems consisting of many spring-mass systems, each with its own natural frequency. Also, each of these systems may have differing degrees of freedom with a differing natural frequency. This collection of possible resonant frequencies, and the many excitation frequencies, all combine to make resonance a very common problem for the transient vibration analyst. Understanding the basics of how a system responds to vibratory forces is important to anyone involved in vibration measurement, analysis, and balancing. From a measurement standpoint, it is important to remember that every object has a resonant frequency...machinery, pickups, brackets, etc. Resonance of a pickup mounting bracket, or the pickup itself, will introduce significant errors to measurements.

RESTRAINING FORCE

The combined effects of the restraining forces of stiffness, damping, and mass determine how a system will respond to a given exciting force. Mathematically the relationship is represented by:

$$M\,a + D\,v + K\,x = M_e\,\omega^2\,e\,\sin(\omega\,t - \theta)$$

For simplification, the above equation can be written as:

Mass term + Damping term + Stiffness Term = Restraining Force

The restraining forces, represented by the various terms in the equation, are what determine how a rotor behaves throughout its operating range. Any excitation force, such as unbalance, is always in equilibrium with the restraining forces of mass, damping, and stiffness. The amount of measured vibration, as a result of these combined forces, will depend upon the combined effect of all three terms in the equation. The phase angle (θ) change as a rotor increases speed and surpasses a resonance region is dependant upon on the relationship between the various terms.

PHASE RELATIONSHIP

To understand the phase relationships of the terms, consider that the mass term is proportional to acceleration, damping term is proportional to velocity, and the stiffness term is proportional to displacement. In equation form, the acceleration term = $-x\,\omega^2\sin(\omega\,t)$ and the velocity term = $x\,\omega\cos(\omega\,t)$. Examining the relationship of the acceleration and velocity equations, a 90° phase difference exists as the terms are integrated. Another integration produces the stiffness term that is proportional to displacement (x) only, and the relationship between the stiffness and damping terms have another 90° phase shift.

The effects of frequency (ω) should also be considered along with the phase shifts noted. Stiffness being proportional to displacement only, and not influenced by frequency, means that essentially the stiffness term is constant throughout all frequency ranges. However, the damping and mass terms are influenced by ω and ω^2, respectively.

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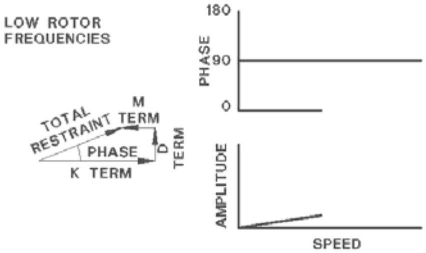
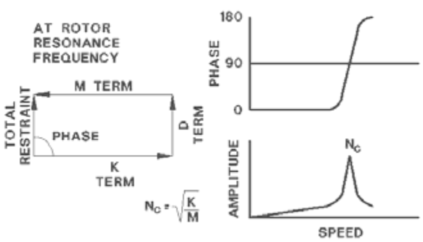
Below Resonance



If each of the individual terms are represented by a vector, and the influences of frequency are included, the result is a type of graph, similar in shape to a triangle. The figure is a graphical representation of the relationships of the terms at low frequencies, i.e. slow rotor speeds. The total restraint vector is the summation of all three-vector terms. Note that the damping and mass terms do not have much influence on the total restraint at low frequencies, leaving the stiffness term as the dominant term. This means that at frequencies below the resonance frequency the rotor behaves as a pure spring, sometimes called a stiff shaft rotor.

At Resonance

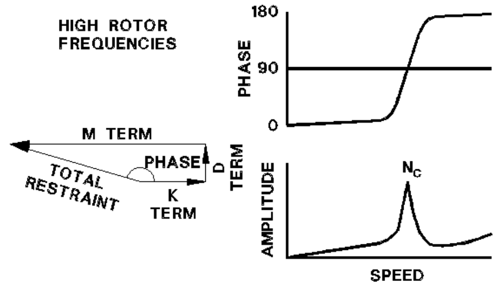


As the rotor frequency increases, the influence of the damping and mass terms become greater due to the influence of ω and ω^2 in the mass and velocity terms. At a certain frequency the stiffness and mass terms cancel each other due to the 180° phase difference in the terms. The figure presents the vectorial relationships and the resultant vibration amplitude response at resonance condition.

When these terms cancel each other the only remaining restraint term is the damping term to control the system vibration. As the stiffness and mass terms approach the point of canceling each other, the system's vibration amplitude will increase to a maximum, constrained only by the available damping from any lubricant present. At resonance the system has lost the restraining forces of the stiffness and mass terms. A machine supported by rolling element bearings, which have little or no damping capabilities, will exhibit a dramatic and sharp increase in vibration amplitude in this region.

This phenomenon is referred to the resonance frequency or "critical" speed. Operation in this zone should be avoided since any change in the available damping can have a dramatic effect upon the measured vibration levels.

Above Resonance



As the rotor frequency continues to increase, the mass term, which is proportional to ω^2, becomes the predominant portion of the total restraint force, growing faster than the other terms. The figure shows the vector representation of the forces and the vibration amplitude at high rotor frequencies. Note that as speed increases further the phase angle change approaches another 90° shift. The rotor behaves as a pure mass with little impact from the constant stiffness term and the relatively slowly changing damping term. A rotor operating in this region is called a flexible rotor since it rotates around its mass centerline, not its geometric centerline.

Thus, as rotor frequencies increase, three regions are found where one of the component terms is dominant over the other two terms. The summation of the three terms is represented by the vector labeled: Total Restraint. The total restraint vector is what is measured as vibration amplitude and its associated phase angle. As the rotor speed passes through each of these regions the measured phase angle will change by 90° and will exhibit an overall phase shift of 180° as it surpasses a critical "resonance" speed: N_c.

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